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SETTLING TIME OF LIQUID IN TANKS UNDER THE ACTION OF MINOR OVERLOADING

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A theoretical model is proposed for determining the time t_0, which is the basis for the concept of ascent and division of large gas bubbles in liquid. The estimate of t_0 given by this model is in qualitative agreement with experiment.

One method of ensuring continuity of a liquid flow pumped out from a tank under reduced gravitation is to apply a small acceleration g to the tank ($g < g_0$). Under the action of this acceleration, the liquid flows toward the intake unit (conventionally, downward) and the bubbles of pressurization gas and liquid vapor which it contains move to the opposite wall of the tank (float upward).

To estimate the time of this process t_0, various models are proposed. In [1], for example, it was recommended that liquid motion be considered by analogy with the free fall of a solid in a field $g \ll g_0$ and that the fall time t_{fall} be determined from the formula

$$S = \frac{g t_{fall}^2}{2}. \tag{1}$$

Experiments [1] show that $t_0 > t_{fall}$, and it was recommended in [1] that the value $t_0 = (2\text{-}5)t_{fall}$ be taken. This means that in t_0, as well as t_{fall}, account is taken of the time of partial damping of the liquid, the time of bubble ascent, and possibly the duration of other processes.

The practical recommendation of [1] as regards determining t_0 is now justified, by considering the ascent of gas bubbles in the liquid under the action a small acceleration $g \simeq (10^{-2})\text{-}(10^{-4})g_0$ rather than the fall of the liquid to the intake unit. It is assumed that the pressurization gas is initially concentrated in a single bubble, which is at the intake unit and then moves to the opposite end of the tank. As the bubble rises, it breaks down into several smaller bubbles, which also, in turn, break down further, thus creating a cluster of bubbles.

To describe the motion of an individual bubble, the semiempirical approach of [2] is used. According to [2], the velocity of steady motion of so-called large bubbles does not depend on their size

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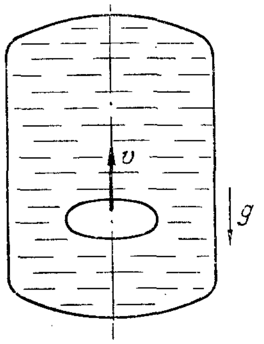

Fig. 1

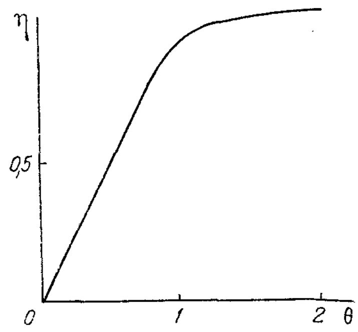

Fig. 2

Fig. 1. Form of ascending large bubble.

Fig. 2. Dimensionless velocity of ascent of large bubbles as a function of dimensionless time.

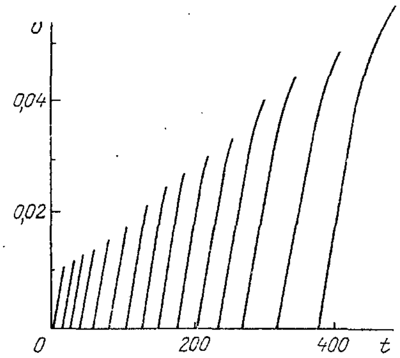

Fig. 3

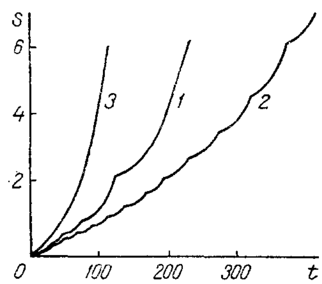

Fig. 4

Fig. 3. Velocity difference of relay of slow bubbles. Each discontinuity in the graph corresponds to the next division of the bubble; v, m/sec; t, sec.

Fig. 4. Path traveled by the relay of slow bubbles when they break down into eight equal parts (curve 1) and into two parts (2). Curve 3 is plotted according to Eq. (1) for the free fall of liquid; S, m.

$$v_* \simeq \left(\frac{4\sigma^2 g}{\alpha \rho_L \mu} \right)^{1/5}. \tag{2}$$

This velocity is established when the Archimedes force and the force of liquid resistance to the bubble motion F_{res} are equal. The formula for F_{res} is

$$F_{res} = \frac{\alpha \mu \rho_L^2 v^5}{4\sigma^2} V. \tag{3}$$

Large bubbles differ from moderate-size bubbles in that they are formed in ascent and take the form shown schematically in Fig. 1, whereas the moderate-size bubbles remain practically spherical. According to [2], the critical bubble radius according to which they may be classified as large or medium (or in other words, of moderate size) is

$$r_{cr} \simeq \left(\frac{324 \mu^2 \sigma}{\rho_L g^2} \right)^{1/5}. \tag{4}$$

When $r > r_{cr}$, the bubbles are said to be large; when $r < r_{cr}$ they are of moderate size. Bubbles of moderate size ascend at a velocity depending on their size as follows

$$v \simeq \frac{1}{9}\,\frac{gr^2}{\nu}. \tag{5}$$

With sudden application of an acceleration g to the liquid, the large bubble begins to rise at an acceleration (disregarding the drag force of the liquid)

$$\frac{dv}{dt} = g\,\frac{\rho_L - \rho_G}{(\rho_L + \xi\rho_L)}. \tag{6}$$

The coefficient of added liquid mass ξ for the bubble considered as a solid body depends on the shape of the bubble, the shape of the tank, and the ratio of their dimensions. For a deformed bubble, ξ is unknown in advance, but is evidently close to $\xi = 1$. In fact, for a rigid sphere immersed in an infinite liquid $\xi = 1/2$. If the tank is regarded as a sphere of radius b and the bubble as an undeformed sphere of radius c, then [3]

$$\xi = \frac{b^3 + 3c^3}{2b^3 - 2c^3}.$$

Then, with a radius ratio $b/c = 1.2\text{-}3$, ξ varies from 2.56 to 0.56. Therefore, $\xi = 1$ may be assumed for rough calculations.

In fact, large bubbles are unstable, and break down into several smaller bubbles as they move. The criterion characterizing their breakdown is

$$r \geqslant r^* = \sqrt[3]{3/K_f}\,\frac{\sigma}{\sqrt[3]{\rho_G\rho_L}}\,\frac{1}{v^2}. \tag{7}$$

According to [2], the drag coefficient of the bubble $K_f \simeq 0.5$.

It is necessary to estimate the time for the bubble to reach a velocity v given by Eq. (7). Assuming that the liquid resistance in Eq. (3) and its inertial force on the bubble (taken into account by ξ) act independently, the equation of motion of a large bubble is written

$$\frac{dv}{dt} = g/\xi - \frac{\alpha\mu\rho_L}{4\xi\sigma^2}v^5 \tag{8}$$

or in dimensionless form

$$d\eta/d\theta = 1 - \eta^5, \tag{9}$$

where $\eta = v/v_*$; $\theta = t/t_*$; $t_* = \xi\frac{v_*}{g}$; v_* is the velocity of steady ascent of the bubble, given by Eq. (2). The solution of Eq. (9) is obtained by numerical integration with the initial condition $\theta = 0$, $\eta = 0$ and is shown in Fig. 2 (assuming $\xi = 1$).

It is unknown in advance how many bubbles are formed from an initial bubble of radius r and how big they are. Therefore, it is assumed that the bubbles divide into two parts of equal volume. Observing the motion of large bubbles shows that, after division, the components move apart, since one takes on a velocity larger than the initial value and the other takes on a smaller velocity. This does not contradict the law of momentum conservation, since there is another component of the process of bubble break down which takes on momentum: the liquid surrounding the bubble, which flows into the gap between the bubbles, retarding one and accelerating the other. After division, the motion of the slower component bubble is followed. On reaching the velocity at which it is unstable, division is again assumed to occur, and the slower component is again followed. This continues until the next bubble in the slow-bubble relay reaches a specified distance from the intake unit (for example, half or all of the tank height). So as to be specific, it is assumed that the next slow bubble begins to rise with zero initial velocity $v = 0$. In this approach to a cluster of bubbles, it is also assumed that other bubbles in the cluster do not influence the motion of the chosen bubble. This is obviously not the case in practice. However, this influence may produce both acceleration and deceleration of the chosen slow bubble. Overall, this influence is averaged over all the bubbles forming the given relay (altogether, $N \simeq 10$-20) and has little influence on t_0. Such division of the bubbles in half results in the greatest ascent time t_0, since the number of division cycles N is a maximum here.

As an example, consider the ascent of a large ($r \simeq 1.5$ m) gas bubble in liquid hydrogen. The parameters of hydrogen are: $\rho_L = 70$ kg/m^3, $\nu = 2 \cdot 10^{-7}$ m^2/sec, $\mu = 1.35 \cdot 10^{-5}$ kg/m•sec, $\sigma = 2 \cdot 10^{-3}$ N/m. Suppose that the bubble is filled with a mixture of helium and gaseous hydrogen at a total pressure of 1 atm; then $\rho_G \simeq 2$ kg/m^3. The result of calculating the velocity difference of the bubbles forming a particular relay (chain) of slow bubbles is shown in Fig. 3, which is plotted for $g = 10^{-4} g_0$. The segments of the graph in Fig. 3 are sections of that in Fig. 2, plotted in the appropriate scale. The points of discontinuity of the graph correspond to successive divisions of the bubbles. In Fig. 4, curve 2 shows the dependence of the relay paths of slow bubbles in the cluster on the ascent time. Curve 1 in Fig. 4 shows that, if the bubble breaks down each time into eight components, the time to reach the specified distance (in the given example, 6 m) is considerably less, since altogether five division cycles occur in this case.

The bubble path when $g = 10^{-3} g_0$ has also been calculated. This case differs in the number of divisions (19 as against 15) and the time scale t_*.

Relating the total time of bubble ascent t_0 to t_{fall} gives $t_0 = 3.5 t_{fall}$ when $g = 10^{-3} g_0$ and $t_0 = 3.6 t_{fall}$ when $g = 10^{-4} g_0$. Thus, the given simple model gives a value of t_0 within the limits recommended in [1].

More precise agreement cannot be expected, since the definition of t_0 itself is somewhat vague and depends on what is regarded as the end of the transient process of liquid motion toward the intake unit.

In fact, bubble breakdown is irregular, and so bubbles of moderate size ($r < r_{cr}$) are also formed; their velocity of ascent is determined by Eq. (5) and is always less than for large bubbles. For example, when $g = 10^{-4} g_0$, in liquid hydrogen, $r_{cr} \simeq 2$ cm. However, this does not influence the estimate of t_0, since bubbles of radius $r \simeq r_{cr}$ do not undergo multiple divisions — according to Eqs. (4) and (7), $r_{cr} < r^*$ — and the total ascent time for these bubbles is found to be no larger than the previously determined t_0 for large bubbles. Overall, small bubbles (for example, with $r < 0.5$ cm), although their ascent time is longer than t_0, do not determine the efficiency of the apparatus pumping out the liquid, especially when capillary intake units are used.

NOTATION

S, displacement of the center of gravity of the liquid, m; g, tank acceleration; g_0, acceleration due to gravity; t_0, time of liquid inflow to intake unit; t_{fall}, time for liquid to fall to the tank bottom; α, dimensionless coefficient, $\alpha \simeq 30$; σ, surface tension; μ, viscosity of liquid; ν, kinematic viscosity of liquid; V, bubble volume; r, bubble radius; r_{cr}, critical bubble radius according to Eq. (4); r^*, critical radius of unstable bubble according to Eq. (7); ρ_G, gas density inside bubble; ρ_L, liquid density; ξ, coefficient of added liquid mass; K_f, drag coefficient of bubble; V_*, velocity of steady bubble ascent according to Eq. (2); η, dimensionless bubble velocity; θ, dimensionless time; t_*, time scale of ascent.

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